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On the Fractal Measurement of Geographical Boundaries

This paper has two main objectives. First, we review and evaluate four different computational methods for measuring the "fractality" of cartographic lines: these are known as the structured walk, the equipaced polygon, the hybrid walk, and the cell count methods. Second, because previous research has restricted the focus of fractal measurement exercises to isolated physical entities, the subject of our empirical study concerns the urban area of Swindon, United Kingdom, that comprises a mosaic of contiguous land-use parcels. In technical terms, the results pinpoint some of the comparative strengths and weaknesses of the four methods, whilst our substantive conclusion is that fractal dimension appears to be a function of both scale and land-use type in our geographical study.

1. INTRODUCTION

For two decades or more, spatial analysis has been dominated by a style of model building which has sought high predictive understanding in numerical terms but has paid little attention to the geometry of spatial form. There has been a tacit assumption that a concern with form is unlikely to yield insights into spatial processes and thus geometry has remained outside of the mainstream of such analyses. The tide, however, is now turning. With the development of computer graphics, the desire for visually plausible model predictions is increasingly easy to indulge, and there is a growing unease that models that are deemed "successful" on the basis of purely numerical and statistical comparisons may not prove acceptable when their predictions are mapped. This development can also be seen in the context of increasing routine supplementation of purely confirmatory model building with exploratory data analysis in which visual displays constitute an integral part of a cumulative model-building process (Everitt and Dunn 1983).

Models are now being developed in which spatial form is considered alongside the exploration of processes that generate such form (Batty and Longley 1986). Such models have implications not just for the manner in which we represent the outcome of spatial process, but also upon how we might develop rule-based strategies for the simulation of such forms. The new geometry of form, pioneered

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by Mandelbrot (1982), which is based on searching out the regular processes which generate irregular shapes known as *fractals*, is central to this quest. Here we describe how this geometry can be used to measure the form of fractal shapes which together make up the mosaic of urban land use. We will argue that this holds the key to an enriched understanding of the cartographic representation of urban structure.

Fractals are geometrical shapes which exhibit the property of self-similarity, a property which has long been recognized in the cartographic profiles of various natural phenomena (Dearnley 1985; Kaye 1978; Mark and Aronson 1984). Measurement and generalization of cartographic lines through an understanding of fractals has a number of advantages over other forms of representation (Muller 1987), leading to the assertion that empirical evaluation of the fractal dimension "may be the most important single parameter of an irregular cartographic feature, just as the arithmetic mean and other measures of central tendency are often used as the most characteristic parameters of a sample" (Goodchild and Mark 1987).

There is no technical reason why fractal measurement in geography should not proceed in a similar manner to applications in particle science, mineralogy, and others, and the technical focus of this paper is to compare four of the most commonly invoked fractal measurement methods. Measurement of geographical boundaries does, however, require the recognition of some important conceptual differences. First, mapping of (human or physical) geographical boundaries usually requires interpolation of continuous surfaces around a series of discrete sample points (Burrough 1986, pp. 120–22) or involves subjective interpretation as to whether the mapped variable (e.g., "built environment") is present or absent in a particular land parcel. The cartographer becomes the arbiter, and arguably also the architect, of the final delineated form. In short, fractal measurements of geographical phenomena must be seen as records of the outcome of various human judgmental processes. Second, no geographical phenomenon is an isolate, since each is embedded within a matrix of contiguous phenomena. Measurements of line character may predominantly reflect the processes that molded the form of either adjacent land parcel, or may reflect both sets of processes in more or less equal amount.

We are not aware of any fractal measurement exercises which have sought to measure cartographic lines explicitly as *boundaries* rather than as *edges* to isolated physical entities. Geographical boundaries permit classification of land parcels by any number of different criteria, such as usage, size, or distance from central business district, and the parcels can be considered for measurement purposes as either complete geographical individuals or amalgams of constituent boundary features. Our intention is to apply four comparable measurement methods to the different juxtaposed land uses in the town of Swindon, which is located in south-central England.

2. EMPIRICAL MEASUREMENT OF FRACTAL CURVES

As is well known, the central fractal concept which permits empirical measurement of curves is that of scale-dependent length. When a line is measured at a base scale r_j, then n_j full steps are necessary to approximate it with a line of length $n_j r_j$: if the line is fractal, when we repeat the process using a smaller step size (r_{j+1}) we will find a disproportionate increase in the number of steps (n_{j+1}) that are necessary to approximate it. Thus for fractal lines, if

$$\frac{r_j}{r_{j+1}} = k, \quad \text{then } \frac{n_{j+1}}{n_j} > k \tag{1}$$

or alternatively,

$$\frac{n_{j+1}}{n_j} = \left[\frac{r_j}{r_{j+1}}\right]^D, \quad 1 < D \leqslant 2, \tag{2}$$

where D is the fractal dimension common to the two measurement scales. The limits defined in (2) define the special cases of the Euclidean straight line (where $D = 1$ and halving the scale yields exactly twice the number of chords) and a two-dimensional plane (where $D = 2$ and halving the scale yields four times the number of chords, the line exhausting the space). Rearranging (2) in a form suitable for statistical prediction gives

$$n_{j+1} = \left(n_j r_j^D\right) r_{j+1}^{-D} = a r_{j+1}^{-D}, \tag{3}$$

where $n_j r_j^D$ acts as a base constant a against which the number of chords n_{j+1} from any interval size r_{j+1} may be predicted. In general,

$$n = ar^{-D}. \tag{4}$$

Taking logarithms of (4) gives

$$\log n = \log a - D \log r. \tag{5}$$

A related equation is based on the perimeter length P, since from (4),

$$P = nr = ar^{(1-D)}, \tag{6}$$

and taking logarithms of (6) gives

$$\log P = \log a + (1 - D)\log r. \tag{7}$$

In the following empirical analysis, D is obtained from (7) by applying simple regression analysis to a series of scale-dependent length measurements. Visual assessment of the fit of the regression line to the series of perimeter-step length observations is made by plotting the log of r against the log of P which produces a so-called Richardson Plot (Richardson 1961). From (7), the slope term of the regression line through a well-behaved scatter of points is clear equal to $(1 - D)$. For the present, we can take "well-behaved" to mean that the points fall very close to a straight line, which is indicative of constant fractal dimension over a wide scale range: this terminology will, however, be reinterpreted in the following empirical discussion when we will consider the possibility of a more general functional specification.

In the remainder of this paper, we will describe four measurement methods and illustrate their application to an empirical measurement problem. The purpose of this is twofold: first, we aim to present some comprehensive assessment of the various fractal measurement techniques which have been used by different authors in different problem contexts, and hence assess their relative technical merits for the assessment of cartographic lines. Repeated analyses of a digitized data set are made in order to make our findings robust. Second, our measurements are depicted on Richardson Plots as a means of visually assessing the use of standard simple linear regression to identify the fractal dimension of curves. An alternative functional specification is suggested in the light of these results and the implications of this are assessed.

3. FOUR MEASUREMENT METHODS COMPARED

Over recent years, researchers in a wide variety of disciplines have developed and applied a range of fractal measurement methods to describe various phenomena which they have investigated: applications are as diverse as particle science (Turcotte 1986), economics (Jensen and Urban 1984) and music (Dodge and Bahn 1986). In most cases, the precise path of the curves is uncontentious and can be taken to be objective for purposes of measurement, the exact resolution of the curves being precisely known. Measurement methods have developed in an ad hoc manner consistent with the particular objectives of these experiments. Spatial analysis can clearly benefit from the adaptation of these different techniques to the widest range of cartographic line measurement problems, for an emerging consensus suggests that indices of "fractality" provide a valuable synthesis of cartographic line structure and character (for a review of the merits of fractal versus other measurement methods, see Buttenfield [1984] and Muller [1987]). Analysts have already begun to attack the measurement task in earnest: however, it is appropriate to evaluate the efficacy of the different measurement methods in view of the range of scales over which geographic phenomena are to be measured, the variety of survey methods (ranging from ground survey to remote-sensing) which are used to generate and codify spatial data bases, the need to retain key line characteristics when storing digitized data bases, and the infinite range of geographical phenomena which might be measured. In this way, the measurement task might become increasingly standardized and routinized as the field develops.

This paper is intended as a contribution towards this task: we will describe and evaluate the use of four different algorithms to measure the fractality of different land-use parcels in Swindon. Our base map comprised a composite hard copy map of urban land use compiled from areal photographs and local land-use maps supplemented and updated using remotely sensed data. This source was compiled for twenty selected towns as part of an unrelated project to assess the energy efficiency of urban built form (see Rickaby [1987] for full details). As such, it is not comparable with any of the standard remote-sensing sources, but it comprises the most consistent and finest-scale data source available to us. Urban land use was classified as being residential, commercial/industrial, educational, transport-related, and public open space (see Figure 1). It was manually digitized (point mode) using in-house software (Bracken, Holdstock, and Martin 1987) to the highest possible level of operator precision. In view of the resolution of the data source, and our previous experience of digitizing and managing coordinate data bases, we estimate the basic level of resolution of the data to be of the order of 50 meters. The four methods used to estimate fractal dimensions are detailed as follows.[1]

3.1. *The Structured Walk Method*

This was originally devised by Richardson (1961) as a manual means of measuring the fractality of an irregular boundary. The method was subsequently automated by Shelberg, Moellering, and Lam (1982), and a similar algorithm was developed for our own empirical investigations. The algorithm essentially simulates Richardson's original procedure of manually walking a pair of dividers around a

[1]Full technical expositions of the algorithms are not reproduced here, but are presented in Longley and Batty (1988). FORTRAN program listings may be obtained from either of the authors, BATTY or LONGLEY, on UK.AC.UWIST.ABCY.TOWP (accessible from the United States via the RUTHERFORD.EARN gateway).

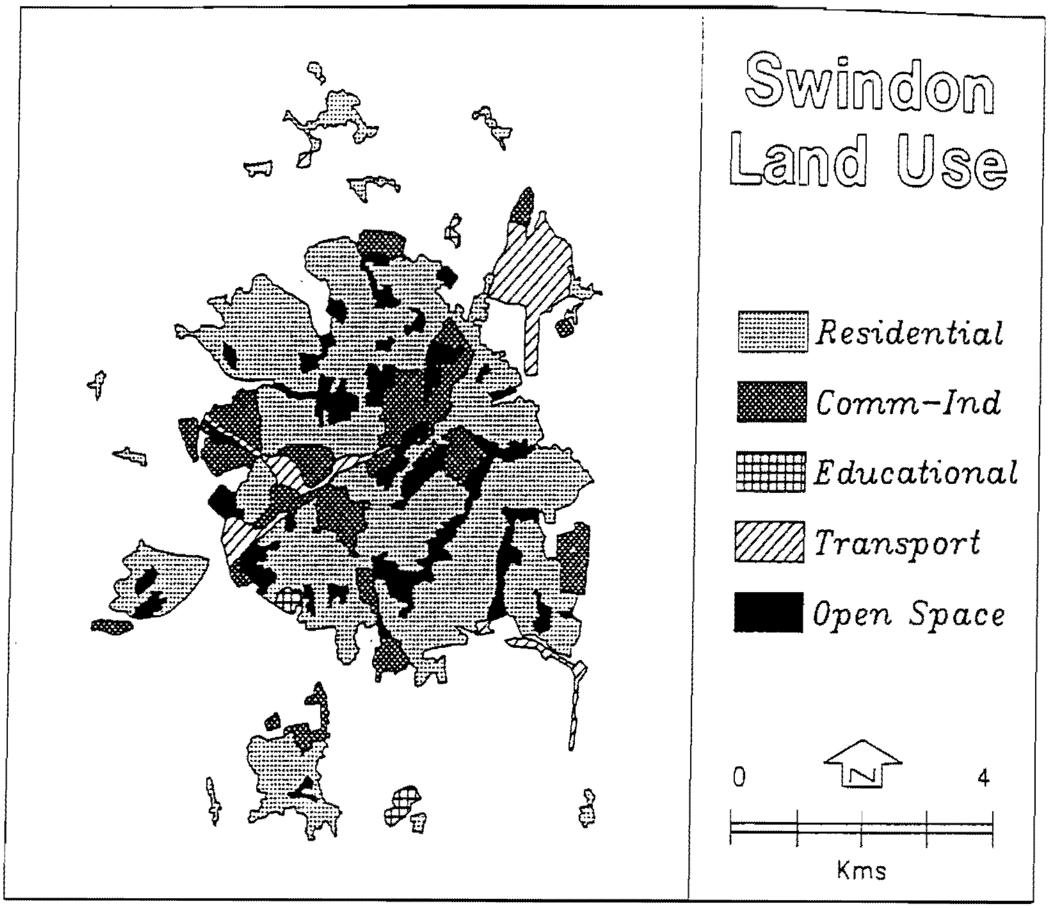

FIG. 1. Land-Use Map of Swindon

cartographic base curve. Where the divider swing intersects the base level curve, a new point is interpolated, and this point constitutes the base point for the next divider swing. For the line of digitized points shown in Figure 2(a), the mechanism is illustrated in Figure 2(b). The object of this exercise is to traverse the curve a number of times with a succession of increasing divider spans and to record the curve length measured at each of these scales. For fractal lines, the measured length will always decrease disproportionately as the divider span is increased, since less and less fractal detail will be picked up as coarser and coarser measurement metrics are used.

Computationally, the intersection of each divider span with the base level curve is calculated using standard trigonometry. In practice, this constitutes a cumbersome computational process, but it is precise since stepped intersections are interpolated exactly onto the base curve. The sequence of span widths is initiated at some proportion of the average distance between successive digitized coordinates on the base curve(s): in our empirical analysis we follow Shelberg et al. (1982) and use an initial divider span set to the average digitizing intensity. Subsequent successive divider spans are ordered in a geometric progression so as to give observations that are equally spaced along the horizontal axis of the Richardson Plots. This creates equally weighted observations in the OLS regression analyses that are used to ascertain fractal dimension. The maximum divider span width was taken to be that which approximates the base curve(s) in fewer than seven chords.

One further point concerns the treatment of "remainder" distances between the last interpolated point and the end of the curve. We adopt the practical procedure of adjusting the final divider span swing to coincide with the end of the curve. This

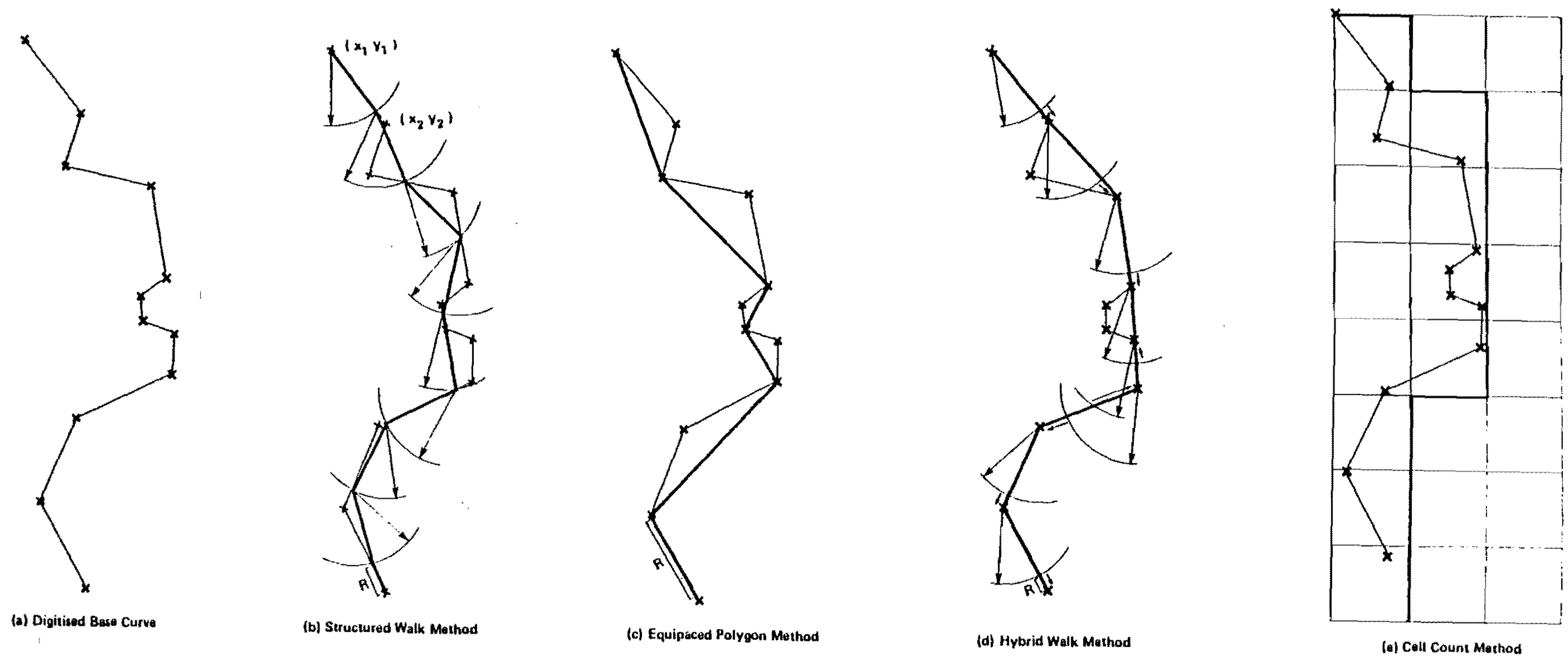

FIG. 2. Illustrative Example of the Mechanisms Underlying the Four Measurement Methods

inevitably injects an element of distortion into the total measured length, and this element attains greater proportional significance at coarser scales. However, the impact of this distortion can be minimized at all levels of resolution by averaging measurements that have been initiated at different points along the digitized base curve. This averaging procedure has previously been used by Kaye (1978) who begins the structured walks from a small number of starting points. In view of the technical agenda for this project, this process was carried to its logical conclusion, i.e., measurements for every single scale change were initiated from *each and every* coordinate along the entire base curve. Our empirical results are thus a reliable statement of the asymptotic efficacy of the measurement method. This procedure also has favorable implications for the realistic measurement of geographical features that are abnormally irregular, since the disruptive effect of either just missing or just picking up such features could create freak measurements when only a single pass through the data is made. The computational implications of averaging as many perimeters as there are points in a data set, however, are enormous as we will note below.

3.2. *The Equipaced Polygon Method*

This was pioneered by Schwarz and Exner (1980) and, unlike the structured walk method, does not require trigonometric interpolation of new points onto the digitized based curve. Instead, changes in resolution are effected by "weeding out" increasing proportions of digitized points at increasingly coarse levels of resolution. The point-weeding proceeds in accordance with a geometric stratified point-sampling interval, i.e., selecting every 1st, 2nd, 4th, 8th, . . . , nth point until the base curve is approximated with fewer than seven chords. Figure 2(c) depicts the result of weeding out every second point from the illustrative base curve. Using this method, there is no way in which any variation in the spacing of the digitized points can be compensated for: thus the stratified point sampling procedure will tend to emphasize those more sinuous stretches of the base curve which will tend to be digitized more intricately, to the detriment of angular promontories where comparatively few coordinates will typically be recorded. (This problem could be circumvented through use of a point intensification algorithm, but only at increased computational costs.) Since no new points are interpolated upon the base curve, we can anticipate that the equipaced polygon algorithm will be less computationally demanding than the structured walk method. The scale observations are displayed on a modified Richardson Plot, in which the horizontal axis records the different point-weeding intervals. In our own empirical work, the same averaging procedures (i.e., using every possible starting point at every scale change) were invoked in order to obtain asymptotically reliable measurements.

3.3. *The Hybrid Walk Method*

This method, developed by Clark (1986) represents something of a compromise between the previous two methods insofar as no new points are interpolated onto the base level curve (unlike the structured walk method), yet the measured perimeter is much less dependent upon the spacing of successive digitized points than is the equipaced polygon method. Essentially the scale of resolution is first used to identify the *digitized* point on the curve in the direction of measurement that is closest to the base point: the straight line distance between the base point and the second point is then recorded and the process repeated using the second point as the new base point, and so on. This procedure is illustrated in Figure 2(d). Because this method is not dependent upon trigonometric interpolation, it is far less demanding of computer processing power than the structured walk method. In our empirical analysis, the averaging procedure described in section 3.1 is used for each and every scale change.

3.4. The Cell Count Method

This method was developed in the work of Goodchild (1980). Scale changes can be thought of as being accomplished by successively throwing a series of increasingly coarse meshes over the digitized base curve: for reasons of comparability in our empirical analysis, the sequence of grid spacings begins with the same initiating step length and then follows the same progression as for the structured walk. This procedure is illustrated in Figure 2(e). In essence, from a given starting point (x_p, y_p) with a given cell size r and direction of traverse, the next coordinate (x_i, y_i) is alighted upon. A test is made to see if this point lies within the same cell by measuring whether $|x_i - x_p| > r$ or $|y_i - y_p| > r$. If either of these conditions holds, a new point is established where the coordinate in question is updated in the direction of greatest increase. Thus if $|x_i - x_p| > |y_i - y_p|$, $x_{p+1} = x_p + r$ and $y_{p+1} = y_p$ whilst if the converse holds, $x_{p+1} = x_p$ and $y_{p+1} = y_p + r$. If the increase along both the x and y axes is less than the grid size r, then a new coordinate point (x_{i+1}, y_{i+1}) is chosen and the tests are made again. A running total is kept of the number of cell sides that have been crossed. It is recognized that this method may be less suited to the task of hugging the more intricate details of the base curve but, because of its low computer processing requirements, it is recommended as a method suitable for yielding a first approximation to the fractal dimension. Unlike the other methods, there is no remainder length at the end of the traverse, since the cell approximation simply finishes when the cell in which the end point exists has been identified.

4. EVALUATION AND COMPARISON OF EMPIRICAL MEASUREMENTS

In substantive terms, an objective of our empirical study was to assess whether different land-use types were characterized by boundaries of distinctive fractality. Since each boundary results when two different land uses meet (or merge), single land-use boundary characteristics are confused by the characteristics of adjacent land uses. Measured patterns may predominantly reflect the processes which fashioned the spatial form of either land use, or may reflect both sets in more or less equal amounts. One of the motives for this evaluation of alternative measurement procedures was to assess whether, in aggregate, it was possible to detect generic differences in the geometry of land-use patterns.

Our digitized map of Swindon comprised a total of 6,059 points and from this five separate files were created. Line segments were allocated to a maximum of two of the five files, contingent upon which two land-use types they separated. The five land-use types were residential, commercial/industrial, educational, transport, and public open space. Thus a line segment separating residential land and public open space would be allocated to both of these files. All segments were thus entered into two files except those that delineated the outer urban boundary, which were allocated only to the interior land use. The number of points allocated to the five files was 2,989 (residential), 1,030 (commercial/industrial), 109 (educational), 510 (transport), and 1,421 (public open space). The comparatively small number of points and segments abutting the educational use, and indeed the small overall area occupied made it impracticable to include this land use in our subsequent analysis.

Computational processing of each of the remaining four land-use files allowed the segments to be reassembled into a series of land-use parcels. With some exceptions, these individual parcels would not yield fractal dimension data over a very wide range of scale changes. For this reason, and for reasons of greater generality of our findings, the parcels of each land-use type were strung together in a sequence by realigning the last coordinate of all but the last parcel with the first coordinate of the next parcel in the land-use file. Figure 3 depicts an example of the composite boundary produced by this procedure. Of course, the principle

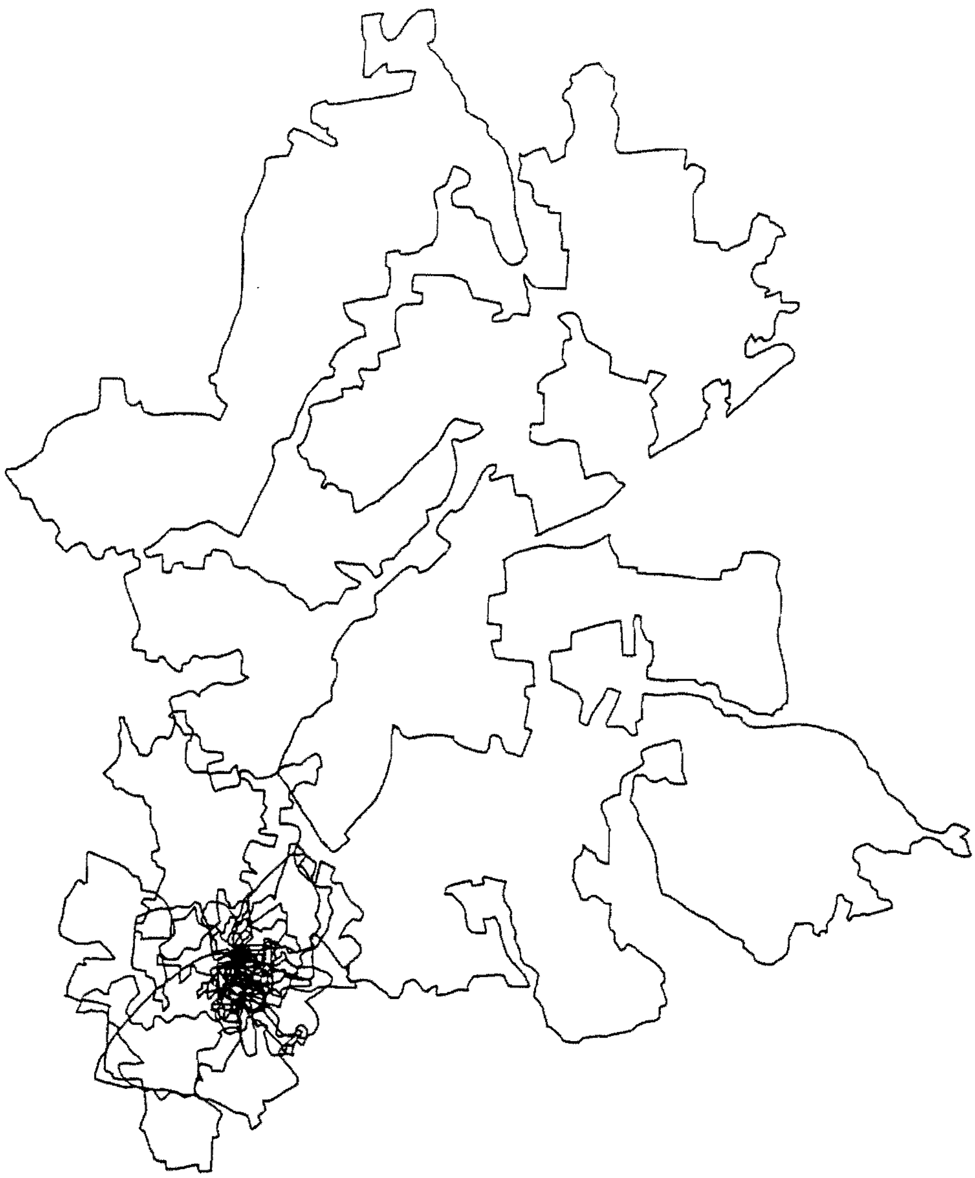

FIG. 3. Example of Realigned Sequence of Land Parcels: Residential Land Use

adopted here is potentially problematic because the chance designation of the first and last coordinates of each land-use parcel effectively determines the orientation of each successive parcel relative to all of its predecessors. This inevitably leads to the loss of spatial and topological information inherent in the boundaries and no sustained investigation was mounted into the effect of alignment method upon measured fractal dimensions. There is scope for further geometrical refinement here, although our own experimentation with an angle-averaging routine did not suggest that the measurement process was unduly sensitive to this phenomenon over the scale range under investigation. Repeated scale-dependent length measurements were made over the scale range specified in section 3 and the results for each measurement method and each land-use type were displayed on Richardson Plots. These are reproduced in Figures 4 through 7.

At this point we should emphasize first that our measurements were made at much finer scale intervals than many previous studies in order that the precise functional form of the scale-perimeter relationship might be accurately identified over the commonly invoked scale range; and second, that repeated measurement of each scale-dependent curve length from every conceivable starting point makes our

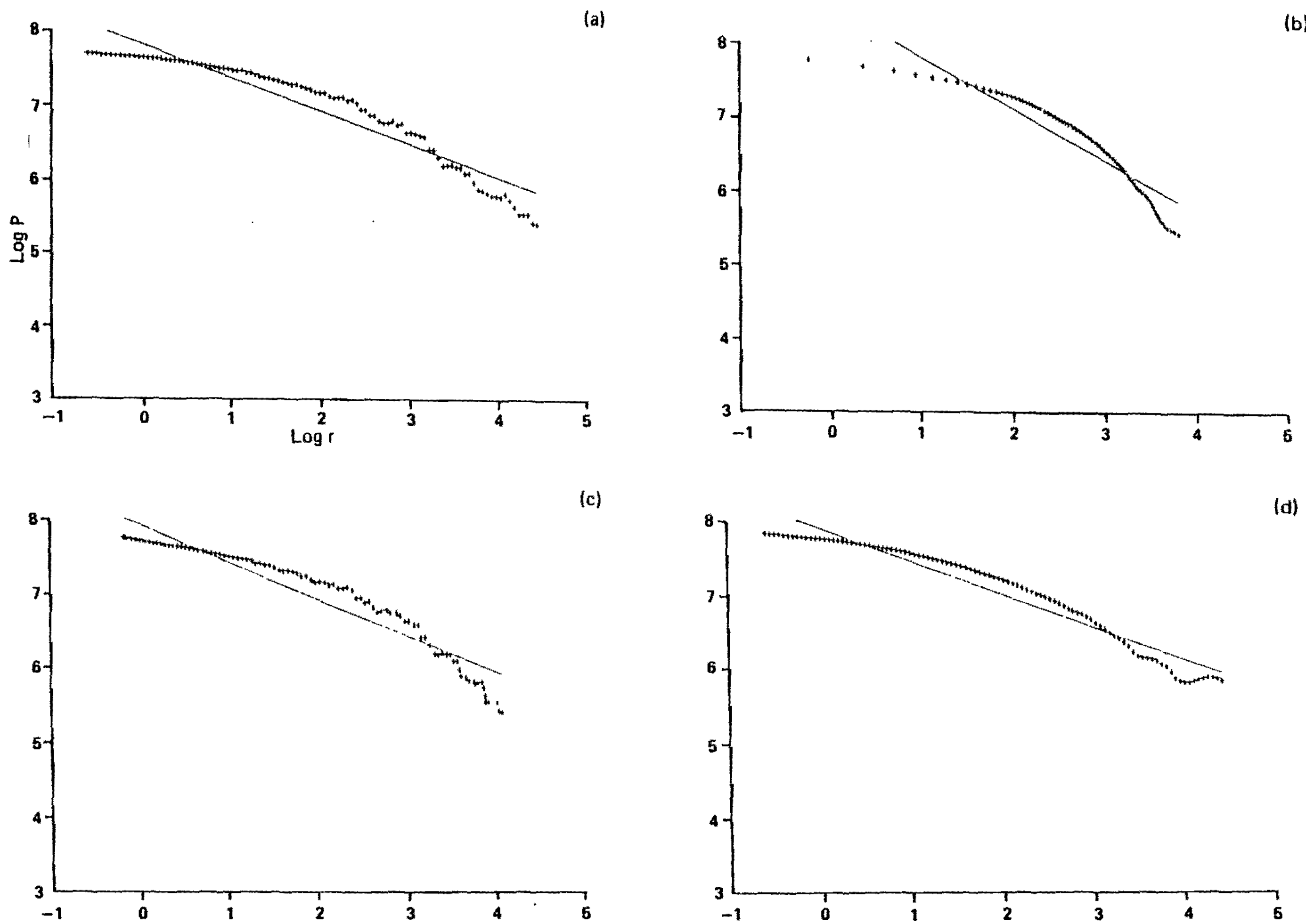

FIG. 4. Richardson Plots of Perimeter-Step Length Relations for Residential Land Use: (a) Structured

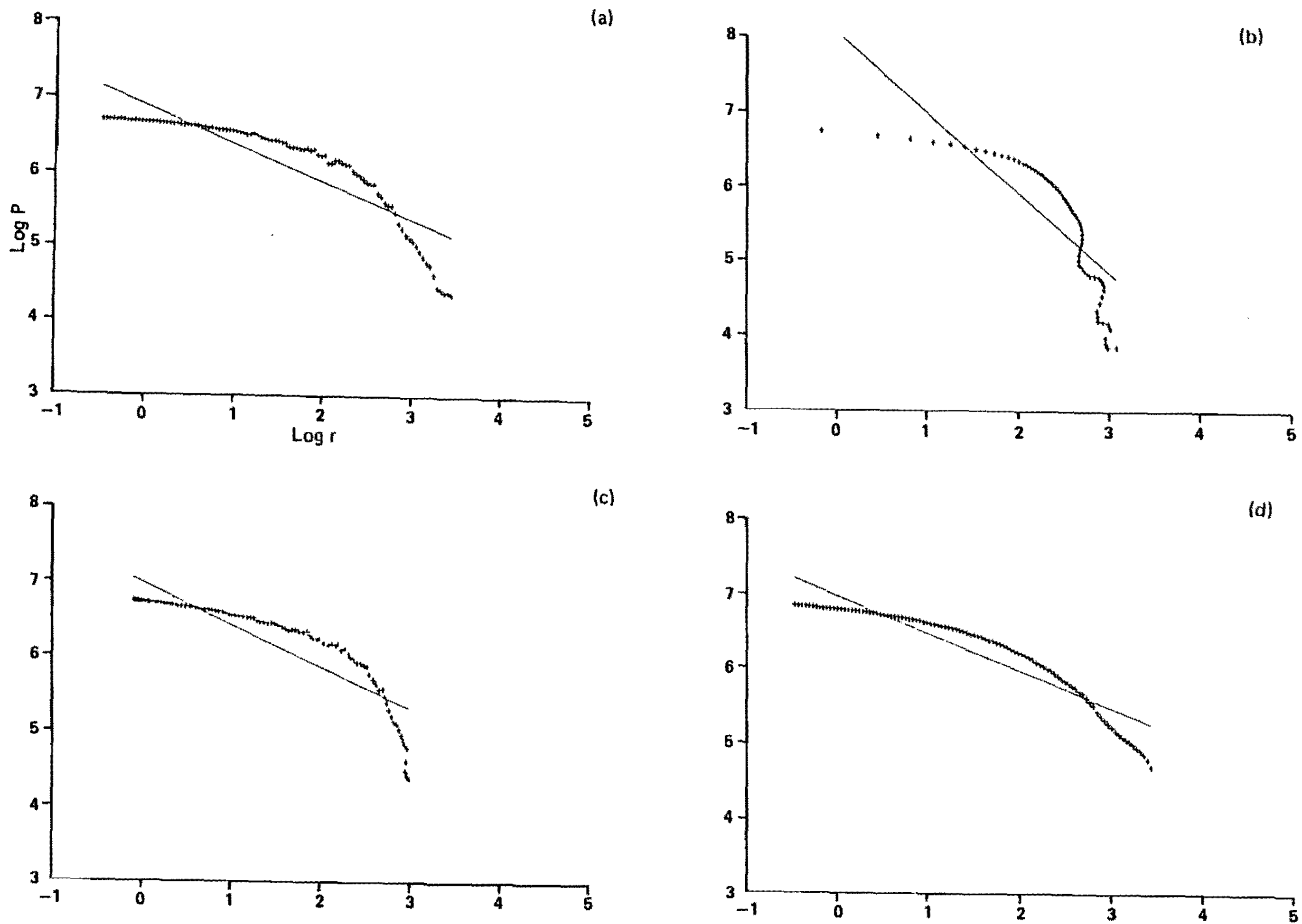

Fig. 5. Richardson Plots of Perimeter-Step Length Relations for Commercial/Industrial Land Use: (a)–(d) as for Figure 4

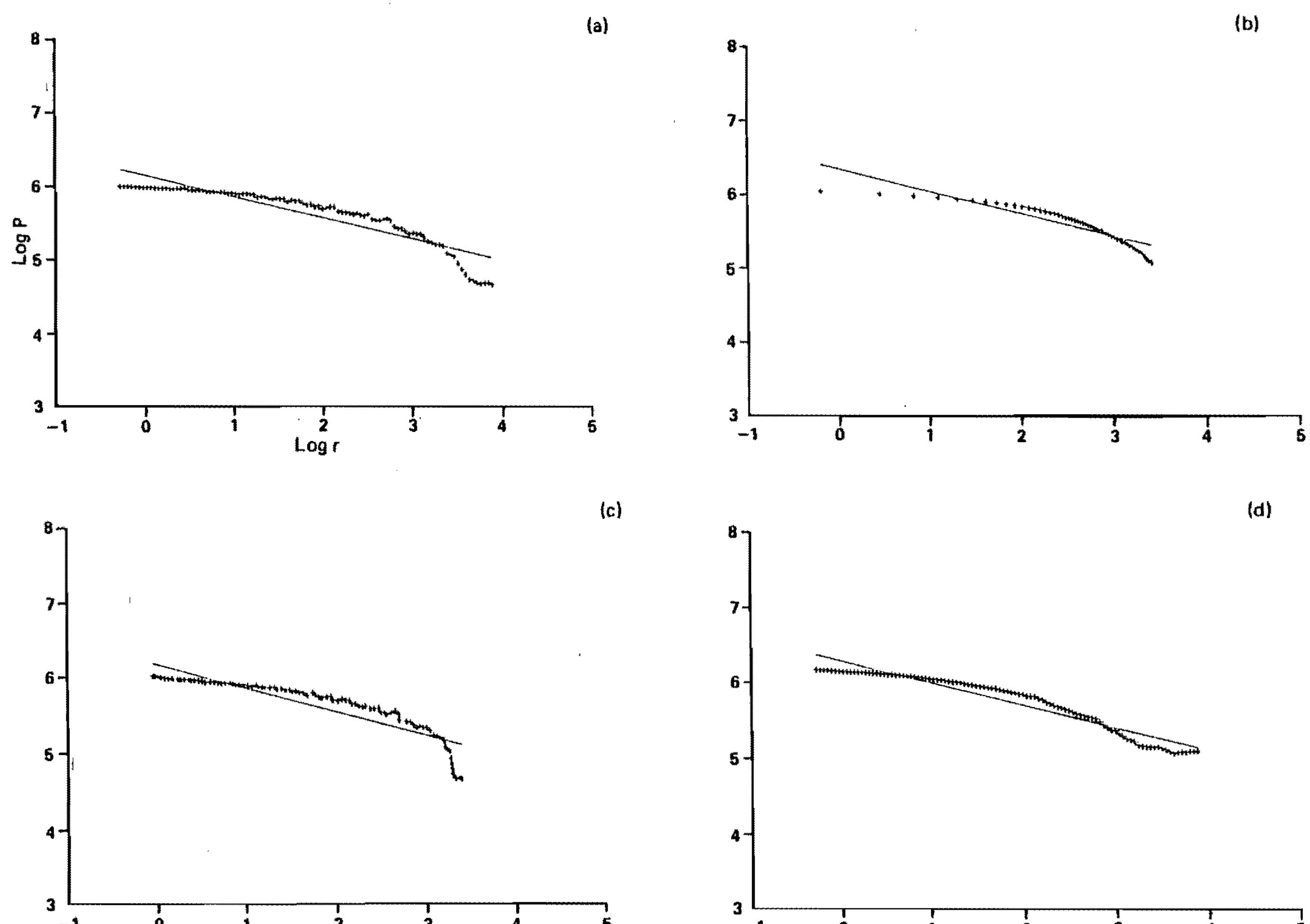

FIG. 6. Richardson Plots of P[illegible] Step Length Relation[illegible] for Transport Land Use: (a) [illegible]

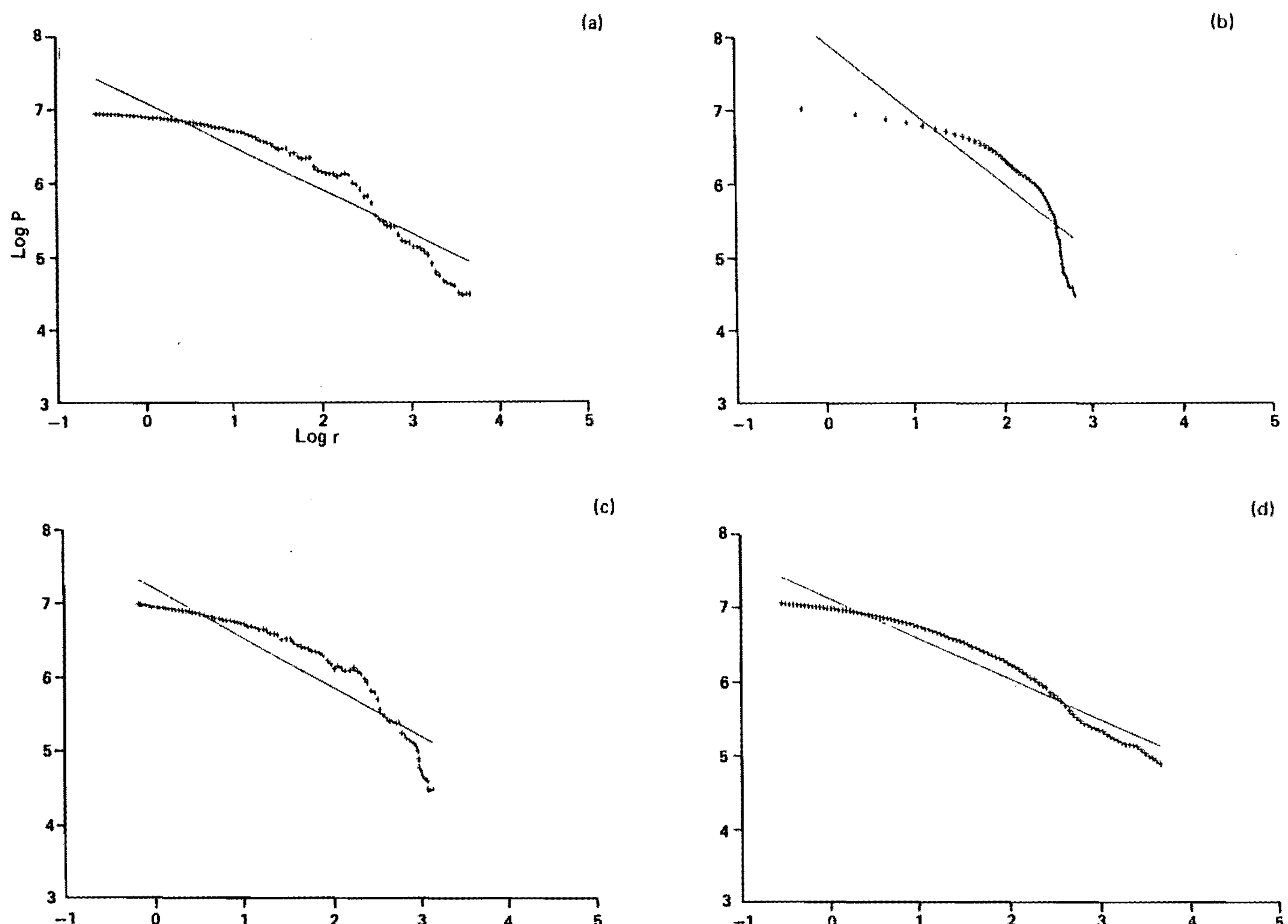
(a)
(b)
(c)
(d)
Log P
Log r
8
7
6
5
4
3
−1
0
1
2
3
4
5

TABLE 1
Regression Analysis of Richardson Plot Data: Standard Formulation, Full Observation Set

		Residential	Commercial/ Industrial	Transport	Public Open Space
Structured walk	a	7.73	6.90	6.13	7.10
	b_1	− 0.337	− 0.508	− 0.290	− 0.593
	R^2	90.5	77.5	83.7	88.2
Equipaced polygon	a	8.50	8.07	6.33	7.93
	b_1	− 0.694	− 1.07	− 3.00	− 0.957
	R^2	85.5	62.5	81.5	69.2
Hybrid walk	a	7.91	6.99	6.17	7.21
	b_1	− 0.496	− 0.566	− 0.314	− 0.666
	R^2	90.5	75.9	80.5	86.1
Cell count	a	7.89	6.96	6.27	7.12
	b_1	− 0.447	− 0.499	− 0.294	− 0.543
	R^2	93.9	87.3	92.1	93.8

final averaged observations more representative of the boundary geometry than those in these other studies. To our knowledge, all previous studies have posited either a constant fractal dimension across all measured scales, have used too few scale changes to reliably interpolate dimensional transitions with increasing scale (Buttenfield 1984), or have subjectively inserted a step function (e.g., Nakano 1984; Schertzer and Lovejoy 1984) into the Richardson Plots in order to accommodate one or more apparent discontinuities in the scatter of points. Our results shown here in Figures 4 through 7 and in a previous study (Longley and Batty 1988) suggest that such interpretations may not be entirely appropriate in geographical examples: rather, they suggest that fractal dimension is itself a function of scale and that the regression fit might be enhanced by the addition of a further scaling constant. Experimentation with a number of different regression lines, purely on the basis of level of statistical fit, has led us to the selection of

$$\log P = \log a + b_1 \log r + b_2 r \log r \tag{8}$$

with the scaling coefficient $(1 - D)$ given by

$$(1 - D) = b_1 + b_2 r . \tag{9}$$

As the scale $r \to 0$ in (9), so $D \to 1 - b_1$. As such, $b_2 r \log r$ acts as a dispersion factor that increases the fractal dimension at increased scales (Longley and Batty [1988] give further details of how this functional form was selected).

The results of fitting the standard simple regression of (7) to the Richardson Plot data are shown in Table 1, and the results of fitting the "multifractal" model in (8) are shown in Table 2. All relevant parameters are statistically significant and, in accordance with established practice (Shelberg et al. 1982), the R^2 statistic has been used as a descriptive indicator of statistical performance. Our results in Tables 1 and 2 show the clear superiority of the multifractal model in R^2 terms. A possible complicating feature highlighted by the Richardson Plots is that the upper end of the scale range appears to be too coarse to yield meaningful measurements, even after the averaging procedure has been used. This is particularly apparent in the case of the cell count Plots. Additionally, commencing the scaled measurements at a fine level of resolution equal to the average digitizing intensity might also give rise to measurements at the finest scales that are confounded by noise attributable to base map and/or digitizing inaccuracies. Although it is not possible to prove definitively, it is felt that such noise is not transmitted throughout the scale range

TABLE 2

Regression Analysis of Richardson Plot Data: "Multifractal" Formulation, Full Observation Set

		Residential	Commercial Industrial	Transport	Public Open Space
Structured walk	a	7.65	6.68	5.99	6.94
	b_1	− 0.171	− 0.0757	− 0.0957	− 0.292
	b_2	− 0.00834	− 0.0212	− 0.00582	− 0.0122
	R^2	99.3	99.5	99.3	98.0
Equipaced polygon	a	7.78	6.47	6.03	6.79
	b_1	− 0.187	0.441	− 0.0453	0.284
	b_2	− 0.0107	− 0.0608	− 0.00750	− 0.0637
	R^2	99.7	94.3	99.9	95.7
Hybrid walk	a	7.74	6.68	5.98	6.96
	b_1	− 0.250	0.0437	− 0.0401	− 0.157
	b_2	− 0.00618	− 0.0365	− 0.0108	− 0.0277
	R^2	99.4	97.7	97.4	99.6
Cell count	a	7.80	6.81	6.20	7.03
	b_1	− 0.0333	− 0.204	− 0.190	− 0.375
	b_2	− 0.00231	− 0.0145	− 0.00310	− 0.00680
	R^2	97.3	99.3	96.8	97.7

because there is general agreement that digitized data are accurate portrayals of shape by the time the data have been aggregated by an order of magnitude.

In order to investigate these two sources of potential confoundment at opposite extremes of the scale ranges, exploratory analysis of "bad data" (high standardized residual and/or high potential leverage values) was carried out. For three of the measurement methods (structured walk, hybrid walk, and cell count), a few points at the *coarsest* end of the scale range were identified as poorly fitting or unduly influencing both the standard log-linear and curvilinear regressions. These observations comprised both high residual and high potential leverage values, and their exclusion from the analysis would be consistent with the view that the range of scales suitable for measuring fractal dimensionality does not extend beyond half the maximum spanning distance between points in the original juxtaposed land-use data. In the case of the equipaced polygon method, the first two observations at the *finest* end of the scale range were diagnosed as outlying observations. Two competing explanations of this phenomenon are plausible. First, we may posit that all the fine-scale measurements (irrespective of the measurement method) are corrupted by short-range noise, and that this is detected only by the equipaced polygon method since these results are most clearly dependent upon the positioning and intensity of the original digitized points. When any of the other three measurement methods is used, the poor fit of the finest scale points is obscured by the interpolation of other noise-ridden observations in accordance with the interpolation of points that would be equally weighted in the regression analyses. The second view is that the outlying observations identified in the equipaced polygon regression are not anomalous in the context of the overall trend in the data.

Regression analyses using both the standard and curvilinear functional forms and omitting the common "bad" data points were performed using all four methods, and the results are presented in Tables 3 and 4. Regarding the omission of points at the coarsest scales for the structured walk, hybrid walk, and cell count methods, we tend to the view that these points did obscure rather than clarify trends in the data. We conclude this because most of the points had high residual values, they appear in the Richardson Plots (Figures 4–7) to disrupt an established trend in the data

TABLE 3
Regression Analysis of Richardson Plot Data: Standard Formulation, Reduced Observation Set

		Residential	Commercial/ Industrial	Transport	Public Open Space
Structured walk	m^*	97 : 100	96 : 100	97 : 100	97 : 100
	a	7.71	6.84	6.11	7.07
	b_1	− 0.312	− 0.440	− 0.266	− 0.563
	R^2	92.3	77.9	83.8	87.5
Equipaced polygon	m	1 : 2	1 : 2	1 : 2	1 : 2
	a	8.79	8.98	6.53	8.60
	b_1	− 0.796	− 1.42	− 0.373	− 1.24
	R^2	90.0	71.4	87.9	76.6
Hybrid walk	m	98 : 100	96 : 100	94 : 100	97 : 100
	a	7.89	6.93	6.12	7.18
	b_1	− 0.477	− 0.492	− 0.263	− 0.626
	R^2	90.7	78.7	85.5	86.7
Cell count	m	97 : 100	97 : 100	97 : 100	97 : 100
	a	7.88	6.93	6.27	7.11
	b_1	− 0.442	− 0.467	− 0.294	− 0.529
	R^2	93.1	87.2	92.1	93.2

NOTE *m denotes the first and last of the range of omitted observations.

TABLE 4
Regression Analysis of Richardson Plot Data: "Multifractal" Formulation, Reduced Observation Set

		Residential	Commercial/ Industrial	Transport	Public Open Space
Structured walk	m^*	97 : 100	96 : 100	97 : 100	97 : 100
	a	7.65	6.68	5.98	6.93
	b_1	− 0.175	− 0.0558	− 0.0824	− 0.252
	b_2	− 0.00805	− 0.0227	− 0.00645	− 0.0148
	R^2	99.2	99.7	99.5	98.5
Equipaced polygon	m	1 : 2	1 : 2	1 : 2	1 : 2
	a	7.84	5.90	6.03	6.30
	b_1	− 0.215	0.794	− 0.0462	0.631
	b_2	− 0.0103	− 0.0698	− 0.00748	− 0.0754
	R^2	99.7	94.7	99.9	96.2
Hybrid walk	m	98 : 100	96 : 100	94 : 100	97 : 100
	a	7.73	6.70	6.00	6.96
	b_1	− 0.238	− 0.0063	− 0.0740	− 0.164
	b_2	− 0.00662	− 0.0317	− 0.00855	− 0.0271
	R^2	99.5	98.6	98.5	99.6
Cell count	m	97 : 100	97 : 100	97 : 100	97 : 100
	a	7.79	6.80	6.20	7.02
	b_1	− 0.296	− 0.178	− 0.190	− 0.344
	b_2	− 0.00355	− 0.0164	− 0.00310	− 0.00877
	R^2	98.4	99.6	96.8	97.9

sets without establishing another one, and their exclusion results in considerable improvement in R^2 values. Regarding the omission of points at the finest scales for the equipaced polygon method, we are, on balance, of the view that these points should not be excluded for three reasons: first, the Richardson Plots depict a smooth curvilinear form throughout the finer scale ranges and the finest scale points appear to be entirely consistent with this; second, every one of the excluded points was a high potential leverage value (only three points had high residual values), yet there was no strong leverage *against* the trends represented in the

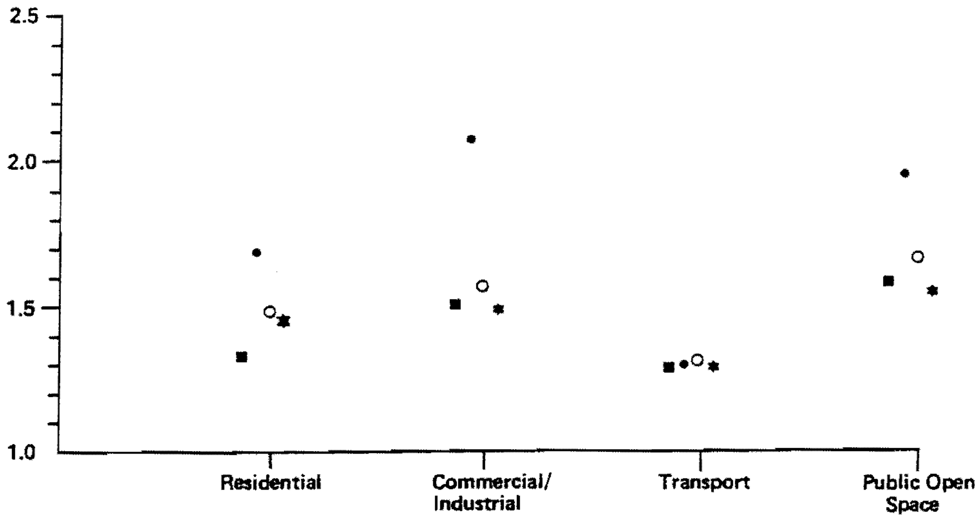

FIG. 8. Graphed Fractal Dimensions Based upon Parameter Estimates from Table 1 ($D = 1 - b_1$)

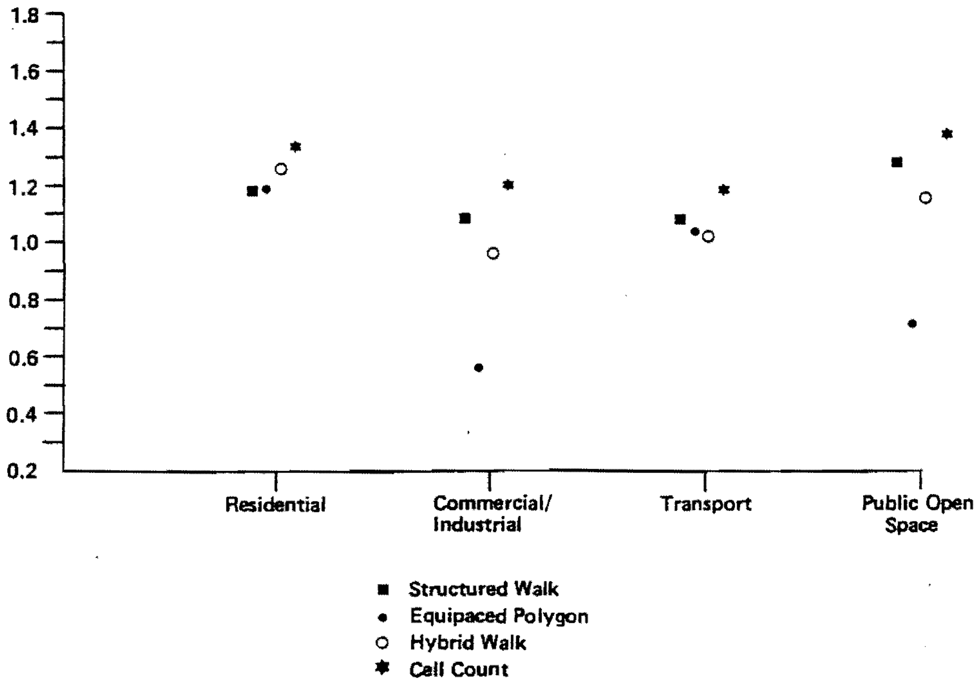

FIG. 9. Graphed Fractal Dimensions Based upon Parameter Estimates from Table 2 ($D = 1 - b_1$)

regressions (i.e., they were identified primarily because of their separation from the other points on the log r axis); and, third, the curvilinear form retains much higher R^2 values.

Our second set of comments concerns the comparative efficacy of the four measurement methods in assessing fractal dimensions of the digitized land-use data. The CPU records for performing the measurements using a MicroVAX II were 5:00:19 (days:hours:minutes) for the structured walk method, 0:01:28 for the equipaced polygon, 3:08:56 for the hybrid walk and 0:19:06 for the cell count: the equipaced polygon and cell count methods are clearly much more frugal than the walk methods in terms of CPU requirements. The parameter estimates from Tables 1–2 and 3–4 are depicted in Figures 8–9 and 10–11 respectively, and show a pleasing degree of conformity between the four methods: differences between the

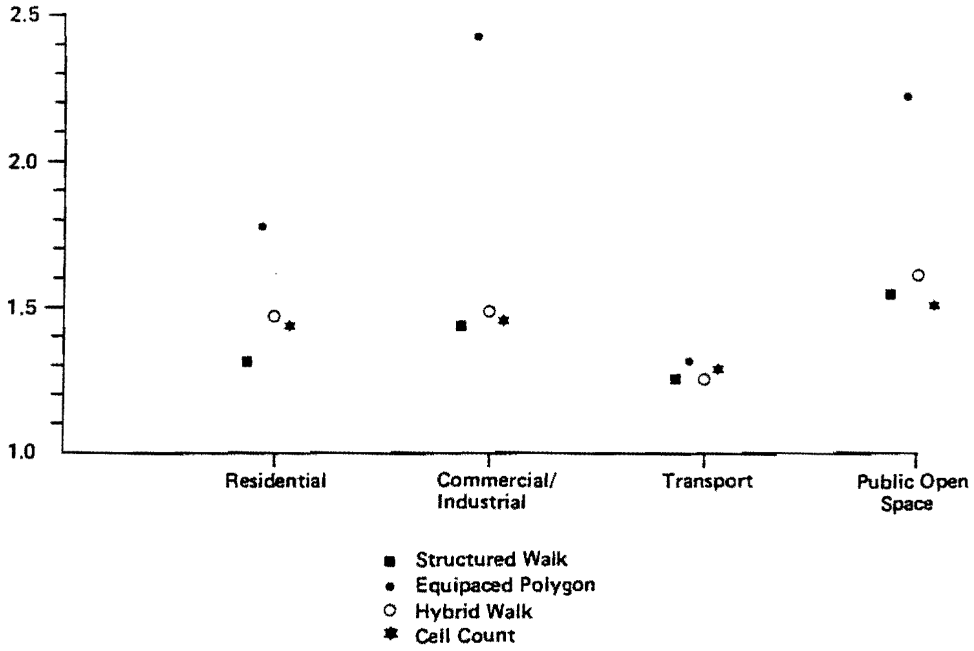

FIG. 10. Graphed Fractal Dimensions Based upon Parameter Estimates from Table 3 ($D = 1 - b_1$)

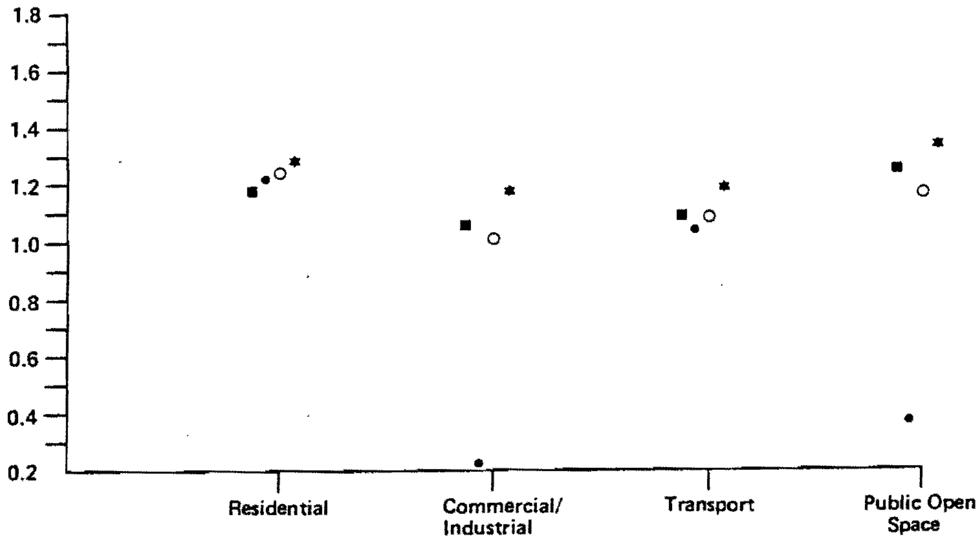

FIG. 11. Graphed Fractal Dimensions Based upon Parameter Estimates from Table 4 ($D = 1 - b_1$)

four land uses far exceed differences attributable to measurement method. This suggests that the cartographic signals characteristic of a particular land-use type effectively overcome the noise generated by each parcel abutting other land uses within the overall land-use matrix. The structured walk, hybrid walk, and cell count methods all produce mutually consistent measurements of the four land-use types, although some degree of comparability breaks down when the equipaced polygon measurements are included (especially since one of the dimensions exceeds that of a surface, i.e., 2). This result should be seen in the light of the authors' previous experience (Longley and Batty 1988) which has highlighted the vulnerability of the equipaced polygon method to the sudden detection of fissures in the base curves between adjacent scale changes. The introduction of the scale parameter in the models used to produce Figure 9 has the effect of dampening the overall range of D values, and one of the hybrid measurements is less than unity: this most probably reflects the convoluted nature of the repackaged boundary, and should probably be seen in this context, rather than as being a shortcoming in the measurement technique per se. Whilst the higher fractal dimensions obtained using the equipaced polygon method are comparable with the other three methods, the lower dimensions are severely dampened beneath unity: we believe this also to reflect the fissure detection problem alluded to above. The mean R^2 value for the standard functional form over the full-scale range using the cell count method is 91.8 percent: this is higher than the values obtained using the structured walk and hybrid walk methods (85.0 percent and 83.3 percent respectively) which in turn is better than the regression fit to the equipaced polygon points (74.7 percent). The mean R^2 values for the multifractal model are mainly above 97 percent for all methods.

5. CONCLUDING REMARKS

The preceding discussion has used a comprehensive and detailed empirical analysis to compare and evaluate four different fractal measurement methods. Our suggestion has been that a timely assault upon the measurement task represents a useful contribution towards the goal of routinizing the treatment of digitized geographical data. The clear result from the standard log-linear regressions is that the differences attributable to the four different land-use types are much more distinctive than differences attributable to measurement technique, even though the distinctiveness of the land-use types is inevitably blurred by the contiguity of the parcels. The three most mutually consistent measurements are recorded by the structured walk, hybrid walk, and cell count methods and, taken together, these suggest that the equipaced polygon method is susceptible to error when the fractal dimensions of different land uses are quite similar. In our previous research (Longley and Batty 1988), we have also suggested that the equipaced polygon method is vulnerable to the spacing of the digitized points, since the sudden detection/omission of fissured segments of the curve can critically alter the recorded dimension at adjacent scale changes if no averaging of observations is carried out. However, the equipaced polygon is the only method likely to flag the dangers of potential over-reliance upon a few observations at small step lengths when evaluating alternative regression functional forms.

Given plummeting computing costs, CP usage is likely to become a marginal consideration in the selection of the most appropriate measurement method, as computed memories and speed continue to increase. However, three of the four methods described here require explicit account to be taken of "remainder" distances, which can introduce considerable distortions into the Richardson Plots at coarse scales. Additionally, our results from the Swindon study clearly suggest that the standard log-linear relationship between step length and perimeter may not be the most appropriate one. Taken together, these considerations suggest a need to undertake considerable averaging of the results obtained at different scales, and the need to identify an unambiguous functional relationship by interpolating a large number of points on the Richardson Plots. In these routine circumstances, CP usage may continue to be an important consideration. Table 5 lists the CP usage in each of our averaged analyses and illustrates how time-consuming the two walk methods are relative to the equipaced polygon method. In view of the results of our analyses, the equipaced polygon method cannot be endorsed, however. A consistent yet computationally frugal approximation to the structured walk is provided by the

TABLE 5
CP Usage on Averaged Analyses: Comparison of the Four Methods

	CP usage (hours:minutes)				
	Residential	Commercial/ Industrial	Transport	Public Open Space	Total
Structured walk	88:24	10:38	2:16	19:01	120:19
Equipaced polygon	1:03	0:08	0:02	0:15	1:28
Hybrid walk	58:54	7:00	1:43	13:19	80:56
Cell count	13:44	1:47	0:24	3:11	19:06

cell count, which typically requires between one-quarter and one-sixth of the CP usage of the walk methods. That said, Figures 4–7 do provide some limited evidence to suggest that the cell count is less capable of recording accurate measurements at the coarsest scales, and our previous experience has been that unaveraged cell count measurements lose consistency in comparison to the walk methods.

Our findings also open a wider debate concerning the notion that fractal dimension is either constant over an entire scale range or that a small discrete number of different dimensions each hold sway over a distinct part of this range. Intensive and repeated measurement over a comprehensive range of scales demonstrates the clear need for some kind of multifractal reformulation of the manner in which scale change occurs. Assessment of the consistency of the multifractal calibrations is more difficult, since the Richardson Plots suggest that the precise functional relationships between step length and perimeter may differ between land-use types. The extent to which it is appropriate to assess a wide range of functional specifications will ultimately depend on the type of regression specification viewed as the most parsimonious means of eliciting fractal dimension. Of course, in a strict sense the curves depicted in Figures 4–7 are not fractal at all. This brings us full circle back to a more profound reconsideration of the geometry of spatial form, and a need to reconceptualize the notion that geographical boundaries are simply frozen in a two-dimensional Euclidean spatial metric. From the measurement standpoint, it may transpire that inductive formulation of the most visually plausible rule for synthesizing the character of land-use boundaries may ultimately advance our insights into those processes that set land-use parcels into the urban mosaic.

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