
Fractal-based description of urban form

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Received 19 January 1986, in revised form 20 May 1986

Abstract. Fractals, shapes with nonintegral or fractional dimension which manifest similar degrees of irregularity over successive scales, are used to produce a consistent measure of the length of irregular curves such as coastlines and urban boundaries. The fractal dimension of such curves is formally introduced, and two computer methods for approximating curve length at different scales—the structured walk and equipaced polygon methods—are outlined. Fractal dimensions can then be calculated by performing log-log regressions of curve length on various scales. These ideas are tested on the urban boundary of Cardiff, and this reveals that the fractal dimension lies between 1.23 and 1.29. The appropriateness of fractal geometry in describing man-made phenomena such as urban form is discussed in the light of these tests, but further research is obviously required into the robustness of the methods used, the relevance of self-similarity to urban development, and the variation in fractal dimension over time and space.

Fractals are irregular shapes whose geometry is scale-dependent. At every scale, the degree of irregularity which characterises the geometry appears the same, this being referred to as self-similarity. At no scale can the form of a fractal or any part thereof be described by a smooth function; thus any such function is said to be nonrectifiable. These properties of self-similarity and nonrectifiability lead to the somewhat startling conclusion that fractals have nonintegral or fractional dimension. Typical examples include rugged boundaries such as coastlines which have dimensions between 1 and 2, and surfaces such as geomorphological landscapes with dimensions between 2 and 3. Fractal objects, however, can exist in any nonintegral dimension and are thus not simply confined to easily perceived natural phenomena (Mandelbrot, 1983).

Coastlines are the most appropriate examples to begin an illustration of these ideas, for the concern of this paper is the application of fractal geometry to urban form, as described by the boundaries of urban areas. Coastlines are obviously self-similar: as one approaches, within the observer's viewpoint which fixes the scale, the degree of irregularity always appears to be the same. At finer scales, irregular detail is a scaled version of that at preceding scales. The consequence of this is the well-known conundrum that the length of the coastline depends upon the scale. At finer and finer scales, the coastline becomes longer and longer, its ultimate length being indeterminate. This fact has been known for many years. For example, Perkal (1958a) refers to measurements of the coast of Istria made by the Viennese geographer Penck in 1894, and there have been several attempts at providing ordered measurement of nonrectifiable curves, such as that initiated by Steinhaus (1954; 1960). Indeed, the conundrum of length is sometimes referred to as Steinhaus's paradox (Nystein, 1966). However, it was Richardson (1961) who was first to describe this phenomenon systematically in formal terms. But he was content to leave the matter as an empirical fact, and it fell to Mandelbrot (1967) to develop the formal framework.

Indeterminate length is the indicator that such lines have nonintegral or fractional dimension. A straight line has dimension 1, a plane dimension 2, and so on; a coastline fills more of the space than a straight line, but does not fill the entire space as does a plane, hence its dimension would appear to be between 1 and 2. There are curves which continually reenter a region, filling space entirely, such as the Peano curve, which have a fractal dimension of 2 but a topological dimension of 1. Thus the more rugged the coastline, the greater the fractal dimension above its topological dimension of 1. In theory, a perfect fractal is formed by a single process which operates repeatedly at all scales, but, in reality, several processes are likely to be at work in moulding the irregularities characterising phenomena such as coastlines. Thus it is widely accepted that fractal dimension is usually constant only over a limited range of scales, and that many phenomena are multifractal. Much present research in applying fractal geometry is therefore concerned with determining the ranges over which particular fractal dimensions apply (Kaye et al, 1985), and, in several disciplines, two distinct ranges of scale described by different fractal dimensions have been identified (Orford and Whalley, 1983).

The purpose of this paper is to apply these ideas to the measurement of irregular boundaries defining urban areas, thus extending fractal geometry from nature to systems which are formed from combinations of man-made and natural processes. Such extensions are not new. Perkal (1958b) has speculated on measuring the urban edge of Wroclaw and several researchers have used fractal measures to define the length of political frontiers, notably Richardson (1961) and latterly Clark (1986a). Although there are many different processes and constraints which mould an urban area, a preliminary investigation into the suitability of fractal geometry seems worthwhile. Moreover, this is a useful complement to our other work, in which land-use patterns are being synthesised from land-use models, with fractal geometry used as a means for displaying realistic-looking maps (Batty and Longley, 1986). Accordingly, we will first formally define the fractal dimension associated with self-similar curves such as coastlines and urban boundaries. Then we will present two methods for measuring the length of such curves at different scales. These involve simulating piecewise approximations to the silhouette defined by the curve, akin to the way one might measure a curve using dividers. These methods are then applied to measurement of the urban boundary of Cardiff by means of digitised data on the extent of the urban area in 1949 and measurements are generated from which the fractal dimension can be computed by using regression. Finally, a brief conclusion as to the appropriateness of these ideas in describing city systems, and themes for future research are presented.

The definition of fractal dimension

Consider an irregular line, X , between two fixed points. Define a scale of resolution, Δx_0 , such that, when this line is approximated by a sequence of contiguous segments or chords each of length Δx_0 , this yields n_0 such chords. Now determine a new scale of resolution, Δx_1 , which is half Δx_0 , that is, $\Delta x_1 = \frac{1}{2}\Delta x_0$. Applying this scale, Δx_1 , to the line yields n_1 chords. If the line is fractal, then it is clear that "halving the interval *always* gives more than twice the number of steps, since more and more of the self-similar detail is picked up" (Mark, 1984, page 293). Formally this means that

$$\frac{n_1}{n_0} > 2, \quad \text{and} \quad \frac{\Delta x_0}{\Delta x_1} = 2. \quad (1)$$

The lengths of the approximated curves or perimeters, in each case, are given as $P_1 = n_1 \Delta x_1$, and $P_0 = n_0 \Delta x_0$. From the assumptions implied in equation (1),

it is easy to show that $P_1 > P_0$, and this provides the formal justification that the length of the line increases without bound, as the chord size or scale Δx converges towards zero.

The relationship in equation (1) can be formally equated if it is assumed that the ratio of the number of chord sizes at *any* two scales is always in constant relation to the ratio of the lengths of the chords. Then

$$\frac{n_1}{n_0} = \left(\frac{\Delta x_0}{\Delta x_1} \right)^D, \quad 1 \leq D \leq 2, \quad (2)$$

where D is defined as the fractal dimension. If halving the scale gives exactly twice the number of chords, then equation (2) implies that $D = 1$, and the line would be straight. If halving the scale gives four times the number of chords, the line would enclose the space and the fractal dimension would be 2. Equation (2) can be rearranged as

$$n_1 = (n_0 \Delta x_0^D) \Delta x_1^{-D} = \lambda \Delta x_1^{-D}, \quad (3)$$

where the term in brackets ($n_0 \Delta x_0^D$) acts as the base constant, λ , in predicting the number of chords, n_1 , from any interval of size Δx_1 relative to this base.

From equations (2) and (3), a number of methods for determining D emerge. Equation (2) suggests that D can be calculated if only two scales are available (Goodchild, 1980). Rearranging equation (2) gives D as

$$\ln \left(\frac{n_1}{n_0} \right) / \ln \left(\frac{\Delta x_0}{\Delta x_1} \right). \quad (4)$$

However, most analyses not only involve a determination of the value of D but also of whether or not the phenomenon in question is fractal, and thus more than two scales are required. Writing equation (3) in more general terms as

$$n = \lambda \Delta x^{-D}, \quad (5)$$

where n is the number of chords associated with any Δx , we can linearise equation (5) as

$$\ln n = \ln \lambda - D \ln \Delta x. \quad (6)$$

Equation (6) can be used as a basis for regression by using estimates of n and Δx from several scales. A related formula involves the length of the curve or perimeter, P , which is given from equation (5) as

$$P = n \Delta x = \lambda \Delta x^{(1-D)}. \quad (7)$$

Equation (7) can be linearised by taking logarithms,

$$\ln P = \ln \lambda - (1 - D) \ln \Delta x, \quad (8)$$

where it is clear that the intercepts in equations (6) and (8) are identical and the slopes are related to the fractal dimension, D , in the manner shown. In subsequent work, we will use equation (8) rather than equation (6), for equation (8) will enable us to check the range of scales used more effectively than equation (6).

Several other methods for determining the fractal dimension of curves have been developed, and these involve approximating line lengths by grid intersections and cell-counts (Goodchild, 1980; Dearnley, 1985; Morse et al, 1985), by area definition of boundaries (Flook, 1978; Mandelbrot, 1983), by area-perimeter relations (Woronow, 1981; Kent and Wong, 1982), and by variograms (Mark and Aronson, 1984). We will not pursue these methods here, but they will be considered in future work.

Simulating scale approximations: the structured walk and equipaced polygon methods
 The original method used by Richardson (1961) to measure the length of coastlines and frontiers involved manually walking a pair of dividers along the boundaries at different scales and then determining D from equation (8). To enable the entire perimeter to be traversed, the last chord length which always finishes at the last coordinate point is generally a fraction of the step size, and the step sizes used at each scale usually reflect orders of magnitude in geometric relationship; that is $\Delta x_n = \alpha^{-n} \Delta x_0$, $\alpha > 1$, which enables each step size to be equally weighted and spaced in the log-log regression. In Richardson's (1961) research, about six orders of magnitude or scales were used which is regarded as sufficient to determine a least-squares regression line.

Computer simulations of Richardson's manual method are now well established. Kaye (1978) refers to the method as a 'structured walk' around the perimeter of an object, and he calls the log-log scatter plot of perimeter lengths versus scale intervals a 'Richardson plot'; this provides a useful visual test of whether or not the phenomenon is fractal. The structured walk method is easy to implement on a computer and here we have used the algorithm developed by Shelby et al (1982) which involves approximating the boundary of an object consisting of line segments between digitised coordinates, with different sized chord lengths. The advantages of this algorithm consist in its adaptation to the display of the various scale approximations on a graphics terminal which enables useful visual assessments to be made of the suitability of the chosen map scales. Here the algorithm is implemented on a MICROVAX II supermini with Autograph X5A display device driven by GINO-F graphics software.

There are two variants involving this method. First, the number of chords and perimeter lengths will depend upon the starting point along the curve. To reduce the arbitrariness of this variation, several workers have suggested the structured walk be started at several different points, and averages of the results then formed (Kent and Wong, 1982). Kaye et al (1985) start the walk at five different points along their curves, but here we have been able to start the walk at each of the $N (= 1558)$ digitised points which define the boundary of Cardiff, the walk proceeding in both directions towards the endpoints of the boundary. The method is extremely time-consuming, and some runs have taken about two and a half hours of CPU (central processor unit) time on the MICROVAX II, with the machine being entirely dedicated to this task.

The second variant involves starting the structured walk at different divider lengths and generating sequences of predictions from these different lengths. The range of scales over which the perimeter lengths were computed varied from about half the average chord length associated with the digitised data, to over the maximum distance between any two coordinate points on the perimeter. The average chord length is computed as follows. First, the distances between each adjacent pair of (x, y) coordinates, i and $i + 1$, are computed from

$$d_{i,i+1} = [(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2]^{1/2}, \quad i = 1, \dots, N-1, \quad (9)$$

and then the perimeter of the digitised base level curve is computed as

$$P = \sum_{i=1}^{N-1} d_{i,i+1}. \quad (10)$$

The average chord length, \bar{d} , of the original curve is therefore given as

$$\bar{d} = \frac{P}{N-1}, \quad (11)$$

and a lower bound for the chord length used to start the approximation, as suggested by Shelby et al (1982), is taken as $\sim \frac{1}{2}\bar{d}$. The maximum distance between any pair of coordinates, which in fine-particle science is referred to as Feret's diameter (Kaye, 1978), is given as

$$F = \max_{i,i+1} \{d_{i,i+1}\} . \quad (12)$$

Kaye (1978), amongst others, suggests that an appropriate upper bound for chord-length approximation is $\sim \frac{1}{2}F$.

The problem with the structured walk method is that, if there are deep fissures in the curve, this can cause perturbations in the Richardson plot through the chord length suddenly falling to the lengths where such detail is picked up. This can then distort the dimension, so an alternative method known as the equipaced polygon method has been suggested by Kaye and Clark (1985). In fact, this method was first implemented by Schwarz and Exner (1980) as an artifact of the particular image analyser used in measuring particle boundaries, and has subsequently been developed by Orford and Whalley (1983). The method involves using the chords associated with the original set of digitised points defining the curve, not new points on the curve as used in the structured walk method. Thus, in the equipaced polygon method, in general the chords are not equal in length. Perimeter lengths are computed by taking the sequence of chord lengths between adjacent coordinates, then between coordinates which are spaced at more than one pair of coordinates apart, and so on. If these sequences are constructed geometrically to ensure more equal weighting in the regressions, the sequences taken would be based on adjacent coordinates, every second, every fourth, every eighth coordinate pair, and so on. For each sequence, an average chord length is computed for use in the log-log regression.

As in the case of the structured walk method, averages of perimeter lengths based on different starting points along the curve reduce the degree of arbitrariness posed by the problem. Apart from the obvious advantage of reducing distortions in the plots and resultant fractal dimensions, the method is easy to compute, for it avoids the time-consuming trigonometry of the structured walk, by using Pythagoras's theorem to compute chord lengths between given points as in equation (9) above. In fact, the method uses about one tenth of the computer time used in each run of the structured walk.

The geometry of urban form: the urban edge of Cardiff, 1949

The boundary marking the extent of the urban area of Cardiff was defined from the 1:25000 Ordnance Survey map published in 1949, as part of a larger project concerned with measuring changes in the urban boundary of Cardiff from 1880 to the present day. The usual problems of definition were encountered in determining the edge of the urban area, and several rules of thumb were invoked. Typically, allotments and other urban fringe land-uses were excluded, villages linked to the urban area by ribbon development were included, man-made alterations to rivers and coast were included, but large landed estates which subsequently become part of the urban fabric were only included if development had surrounded them. The entire definition process emphasised the obvious problems that urban processes and constraints operate at different scales, thus throwing some doubt on the fractal concept of self-similarity in this context; but perhaps no more doubt than exists in other areas of the physical sciences where fractal concepts have been shown to apply only over restricted scales. Nevertheless, the complex concatenation of

processes and constraints which form the urban area does lead to visual forms which, superficially at least, appear fractal.

Once the boundary had been defined, it was digitised to within 1 mm resolution; the coastline contained some 900 points, whereas the urban boundary was based on 1558 points. Figure 1 shows the digitised outlines as well as a coarse approximation to the boundary produced by the structured walk method, which is about thirty times the scale of the original data. Note the way the approximating polygon touches the original boundary, and note the equal chord sizes, apart from the last chord. The coordinates were initially processed with the MICROPLOT software (Bracken, 1985) in which data files are first created on a BBC Micro and then subsequently cleaned up and transferred to the MICROVAX II where all the simulation was done.

The perimeter of the digitised boundary determined from equations (9) and (10) gave $P = 3104.456$ units, with the average chord length $\bar{d} = 1.993$, from equation (11), and the Feret diameter $F = 432.935$, from equation (12). These measures are useful to keep in mind when we discuss the applications of the structured walk and equipaced polygon methods. We will deal first with the structured walk method. For a given chord length used to start the sequence of predictions of perimeter lengths, a complete series of ten chord lengths are used in the approximations, starting from the finest level of scale now given by Δx_0 and moving to coarser scales, Δx_n . The sequence of chord lengths is computed from $\Delta x_n = 2^n \{\delta\}$ ($n = 0, 1, \dots, 9$) where δ is the start length which is always a function of \bar{d} , the average chord length. Thus, for example, where $\delta = \frac{1}{4}\bar{d}$, which is the lower bound recommended by Shelby et al (1982), the sequence of chord lengths used are in the following ratios $\{\frac{1}{4}, 1, 2, 4, 8, 16, 32, 64, 128, 256\}$. In this case, $\Delta x_0 \sim 1$, and $\Delta x_9 \sim 510$ which is much larger than Kaye's (1978) upper bound of $\frac{1}{4}F$. To provide some feel for this range of approximations, we have plotted the approximated boundaries of Cardiff for Δx_n ($n = 0, 1, \dots, 8$) in figure 2. With Δx_9 , the boundary is approximated by a single chord, which is clearly inappropriate. Indeed, even with Δx_7 and Δx_8 , the approximations are too coarse to be of much use. This is clear from

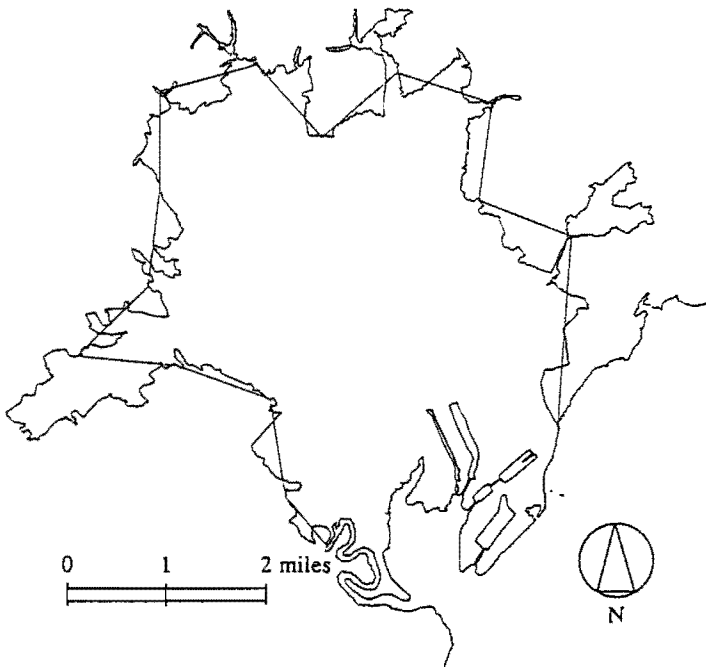


Figure 1. The digitised urban edge of Cardiff in 1949 and a typical scale approximation.

figure 2 which shows that this kind of visual test is essential in selecting an appropriate range of measurements for use in the subsequent regressions.

Our experience in using the structured walk method involved selecting ten different starting values of the chord length δ , and generating ten sets of measurements for each of these starting values. The values of δ chosen were $\delta = 0.4\bar{d}, 0.5\bar{d}, 0.6\bar{d}, 0.8\bar{d}, \bar{d}, 1.5\bar{d}, 2\bar{d}, 3\bar{d}, 4\bar{d}$, and $5\bar{d}$. From the sequences generated, it is clear that several of the chosen measurements are the same between series, but each of the regressions developed below involves different sets of measures. Before we turn to these results, a visual comparison of each of the ten sequences generated is contained in the Richardson plots in figure 3 which show the ten measures of $\ln P$ versus $\ln \Delta x$ for each of the ten starting values of δ . These plots are all on the same scale for comparative purposes and also show the values of $\pm \bar{d}$, \bar{d} , $\pm F$, and F .

At this point, it is appropriate to describe how the equipaced polygon method has been applied. The Richardson plot associated with this method is also shown

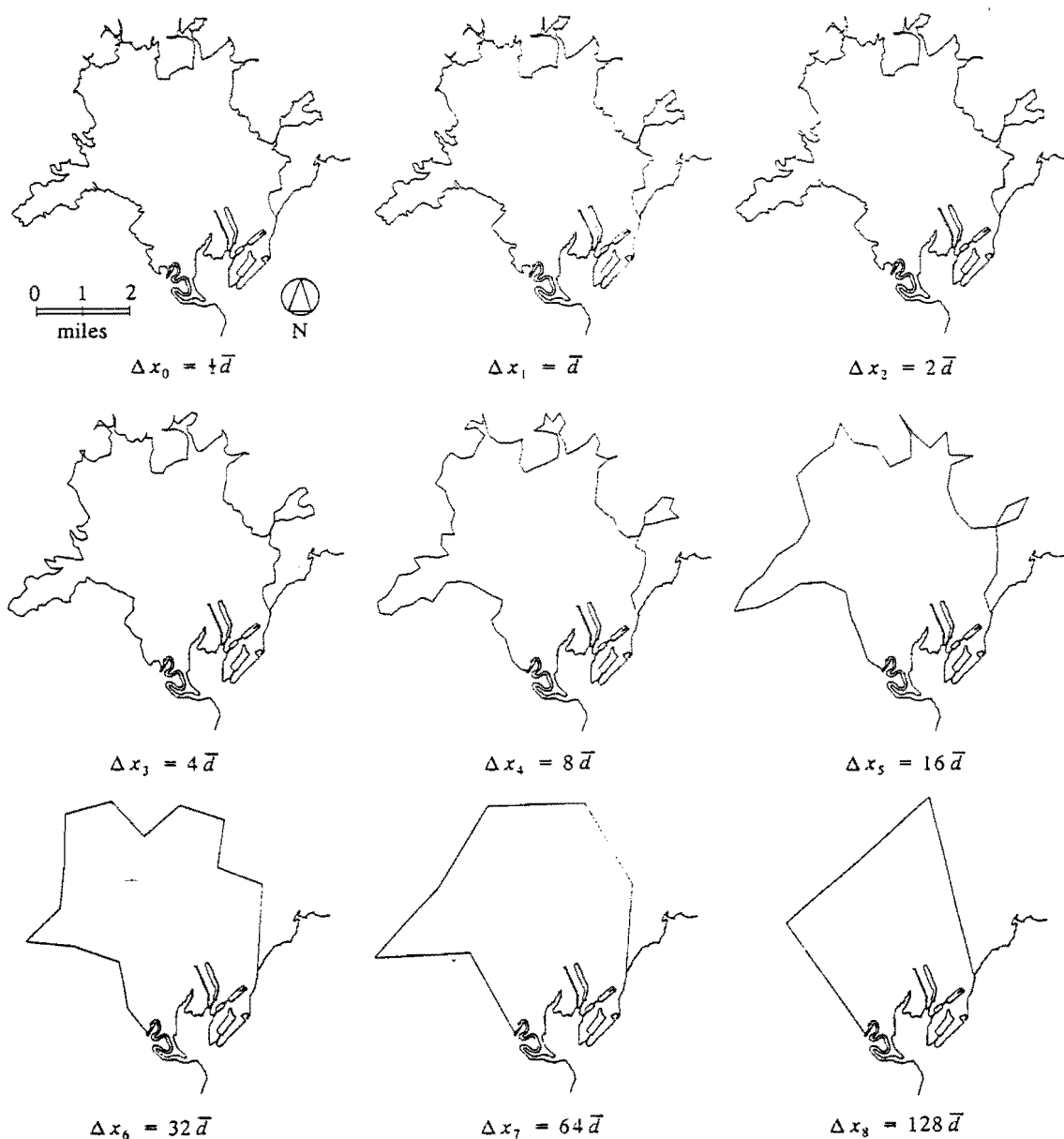


Figure 2. A sequence of scale approximations to the urban edge of Cardiff.

as the last plot in figure 3. Whereas the chord lengths in the structured walk are in geometric relation, it is the chord paces between the given digitised points defining the boundary which are geometrically ordered. The sequence of ten chord lengths is as follows. The initial chord length, Δx_0 , is computed as the average of all single-step segments; the second length, Δx_1 , the average of all two-step segments; the third length, Δx_2 , the average of all four-step segments; and so on, with the rest of the sequence being provided by 8, 16, 32, 64, 128, 256, and 512 step segments. With 1558 coordinates, the last set based on 512-step segments gives just over three chords which from figure 2 is not a good approximation. In fact the process is a little more complicated than this: for all steps over the single step length, averages are computed from alternative starting points to remove the arbitrariness of always starting with the first set of coordinates.

Before we consider the results of the regressions, we now need to consider how we can systematically narrow the range of results we are able to generate, and to

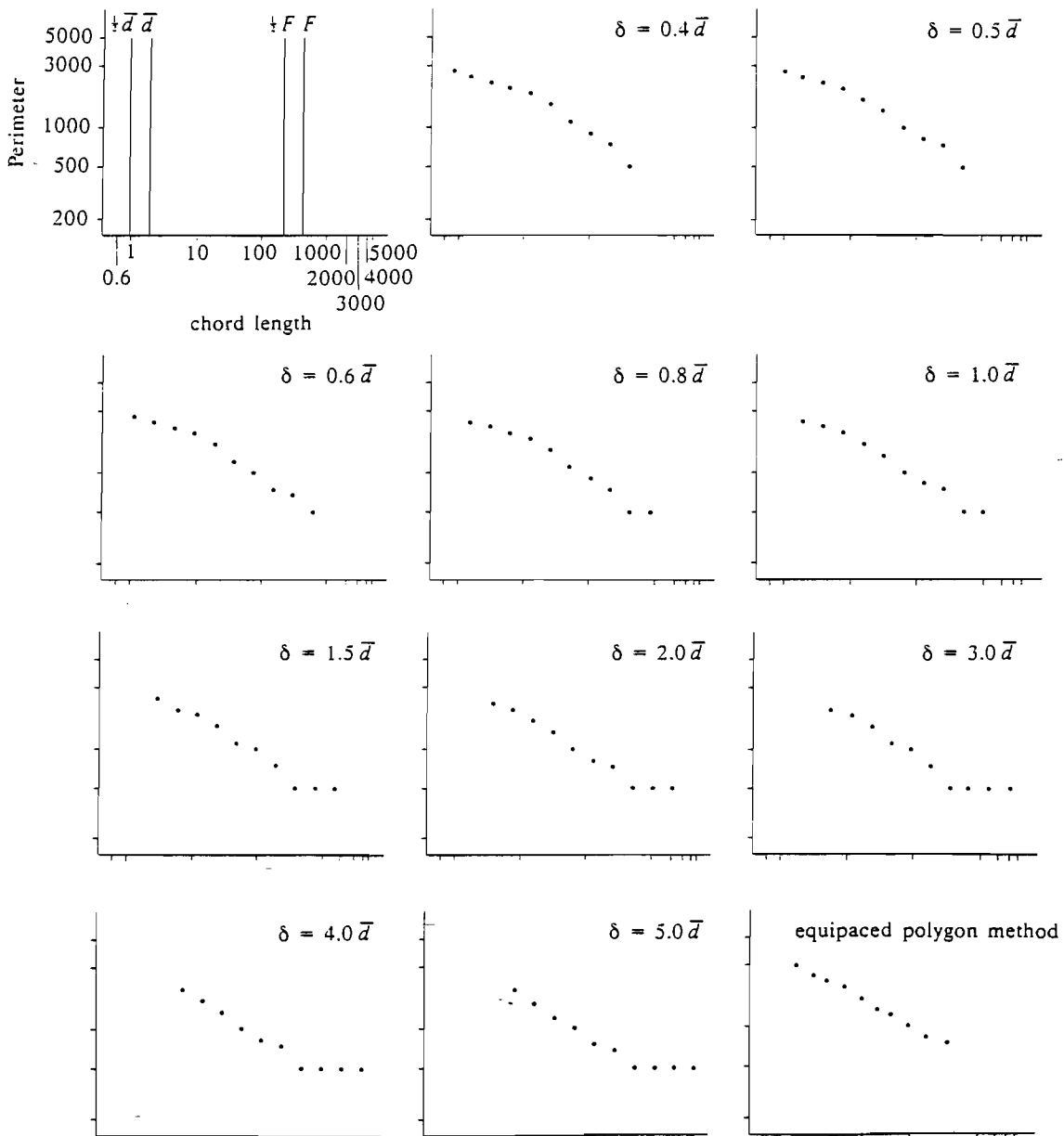


Figure 3. Richardson plots based on ten structured walks and an equipaced polygon approximation.

this end, we have used five criteria. First, we have used the range $0.4\bar{d} \leq \Delta x \leq 0.5F$ to select those observations which are appropriate. Second, we have used the Richardson plots to identify outliers for exclusion. In particular, when $\Delta x > F$, then the algorithm always gives the same perimeter length because it always closes the single chord on the last coordinate point. Such points show up horizontally on the plots and must be excluded. Third, the scale approximation must be acceptable visually. An examination of figure 2 suggests that approximations with ten chords or less are not satisfactory in representing the overall shape, and thus must be excluded. Fourth, the r^2 measure of fit should always be better than 0.95, and, fifth, the standardised variation in average perimeter length for each Δx should not be greater than 10% of the mean value. This also enables poor approximations to be excluded.

For each of the ten starting values of δ in the structured walk method, and for the equipaced polygon method, we have performed regressions on all ten points shown in the Richardson plots in figure 3, on the first 9, the first 8, 7, 6, then 5, below which it is not appropriate to carry out such least-squares fitting. The absolute values of the slopes of the regression lines $|\beta| = (1 - D)$ are shown in table 1 along with the r^2 values, but, as Shelby et al (1982) indicate, such r^2 values should not be used to assess goodness of fit in the strict statistical sense. In table 1, the figures which are in bold type involve regressions in which the observations meet all the five criteria mentioned above, and this narrows the range considerably. Note that the fractal dimension is given by adding 1 to the absolute slopes in table 1, that is, $D = 1 + |\beta|$.

Table 1. Slope and r^2 values from regressions of the data shown in the Richardson plots in figure 3.

Starting values (δ) ^a	Number of 'observations' ^b					
	10	9	8	7	6	5
0.4 \bar{d}	0.269 (0.953)	0.244 (0.959)	0.231 (0.947)	0.207 (0.944)	0.177 (0.961)	0.155 (0.969)
0.5 \bar{d}	0.278 (0.969)	0.258 (0.975)	0.255 (0.963)	0.236 (0.956)	0.211 (0.956)	0.180 (0.975)
0.6 \bar{d}	0.279 (0.975)	0.263 (0.975)	0.254 (0.964)	0.236 (0.963)	0.216 (0.953)	0.185 (0.966)
0.8 \bar{d}	0.292 (0.976)	0.291 (0.966)	0.266 (0.973)	0.254 (0.962)	0.231 (0.957)	0.198 (0.974)
\bar{d}	0.291 (0.982)	0.297 (0.977)	0.276 (0.983)	0.278 (0.975)	0.261 (0.967)	0.234 (0.963)
1.5 \bar{d}	0.293 (0.975)	0.309 (0.980)	0.308 (0.971)	0.280 (0.980)	0.261 (0.980)	0.254 (0.963)
2.0 \bar{d}	0.282 (0.969)	0.304 (0.984)	0.315 (0.982)	0.293 (0.989)	0.303 (0.987)	0.289 (0.979)
3.0 \bar{d}	0.274 (0.945)	0.303 (0.972)	0.327 (0.984)	0.331 (0.977)	0.301 (0.986)	0.282 (0.985)
4.0 \bar{d}	0.254 (0.924)	0.284 (0.958)	0.313 (0.981)	0.331 (0.983)	0.306 (0.989)	0.328 (0.996)
5.0 \bar{d}	0.245 (0.915)	0.276 (0.953)	0.308 (0.984)	0.331 (0.996)	0.321 (0.997)	0.329 (0.996)
Equipaced polygon method	0.285 (0.993)	0.294 (0.994)	0.285 (0.994)	0.275 (0.995)	0.265 (0.996)	0.253 (0.998)

^a Starting values in each sequence of the structured walks.

^b Number of 'observations' of perimeter-chord lengths used in regressions. The first value in each row-column is slope $|\beta|$; the second value in parentheses is r^2 .

For the structured walk method, there is still a large variation in fractal dimension from $1.155 \leq D \leq 1.289$ and from table 1, it is quite clear that as finer and finer scales come to dominate the regression, the lower the value of D . This implies that there is greater irregularity at coarser scales, but it may also indicate that where the scale is below the level of resolution of the digitised boundary, that is, where $\Delta x < \bar{d}$, then no further detail is picked up and the boundary must be considered Euclidean. This is the case for the first four starting values (first four lines) in table 1, and, if these are excluded from consideration, the range of D is from 1.234 to 1.289. In fact, the rule of thumb suggested by Shelby et al (1982) that δ should begin at about $\frac{1}{2}\bar{d}$ should be reevaluated in future work, so that the variation around \bar{d} can be considered.

The equipaced polygon method gives much more acceptable results in these terms, for the basic level of detail sampled is no lower than the digitised base level. In fact, the fractal dimensions produced are much more stable than those generated by the structured walk method. These range over $1.253 \leq D \leq 1.285$ for regressions involving the first eight or fewer values. Moreover, the Richardson plot in figure 3 is the best of those shown there in terms of r^2 performance and linearity. We have also run the equipaced polygon method for up to to five hundred chord sizes, where the increase in step length between coordinates is based on an arithmetical increase, that is 1, 2, 3, 4, 5, ... step lengths. In this case, the range of fractal dimensions from the first ten to the whole five hundred chord sizes varied from 1.249 to 1.327 which is a much narrower range than that shown in table 1 for the structured walk applications.

Conclusions

The fractal dimensions of a wide range of physical phenomena have been studied (Burrough, 1984), but rarely has the irregularity of artificial boundaries been investigated. Richardson's (1961) results on the German and Spanish-Portuguese frontiers, which produced values of D of 1.15 and 1.14, respectively, are the closest examples to the ideas suggested here. The range of variation in fractal dimension for coastlines, however, is quite wide, Richardson's (1961) result of $D = 1.25$ for the west coast of Britain, corrected by Shelby et al (1982) to 1.267, which inspired Mandelbrot (1983) to suggest that the Koch curve with $D \sim 1.262$ as a good model for a coastline, seems to be towards the upper limit.

The range of variation recorded in table 1, from $1.15 \leq D \leq 1.33$ which can be narrowed to $1.23 \leq D \leq 1.29$, seems to be largely an artifact of the methods used, and it remains to be seen whether or not the values calculated here are characteristic of other cities.

This range of variation in D associated with different methods is not unusual (Burrough, 1984), but it does hasten the search for methods such as the equipaced polygon method which are more robust than the structured walk. A method which combines the advantages of both has been proposed by Clark (1986b), and his tests appear promising. Cell-counting methods of the sort used by Goodchild (1982) are also worth exploring, although preliminary tests on this data set reveal that these are considerably less accurate than the walking or pacing methods. It is also clear that appropriate selection of the sequence of scale approximations, and the reduction of any arbitrary variation through averaging are essential in producing appropriate D values.

Finally, the processes which structure urban form and urban edges should be investigated further with respect to the types of irregularity characterising different cities in time and space. This might be accomplished by having regard to what urban theory suggests about the concatenation of processes, but also by recognising

different types of irregularity at different scales and over different sections of the same phenomenon. Historical variations in fractal dimension are also likely, for the development of cities has been influenced by processes whose form and scale has changed over time. In the quest to synthesise overall land-use patterns which fully characterise urban form and structure, research needs to be started on measuring the degree of irregularity in patterns whose parts are disjoint. These are all themes for future research.

Acknowledgements. Beryl Collins typed the original manuscript, Tracy Dinnick drew the diagrams, and Ian Bracken advised on the digitisation of map boundaries. This paper was first presented at a Special Session on Quantitative Residential, Retail and Migration Models, at the Institute of British Geographers Annual Conference, University of Reading, Berks, 7 January 1986.

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